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13. ABSTRACT

This report describes a method for objectively calculating the maximum (limiting) probability that predictions, made by standard quantitative techniques, will succeed. The method presented is based on specialized probability theory but the theory is not emphasized here. Rather, calculations of quantities set forth in this report are explained and interpreted with examples of application.

Such limiting probabilities are concluded to be valuable in determining the reliability of results produced by given forecast techniques.

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OPS

94 - Tech Lit

A GENERAL METHOD
of
PROBABILITY PREDICTIONS
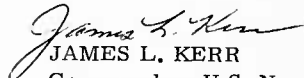## in
SYNOPTIC METEOROLOGY

FOREWORD

This publication was prepared under Task 9, "Forecasting Rules for Specified Strategic and Tactical Areas," and describes a method for obtaining the maximum or limiting probabilities for predictions made by standard quantitative techniques. The method has a theoretical basis, but this will not be emphasized here. Instead, the calculations of the quantities presented in this report are explained and interpreted, with examples of how they can be applied.

The limiting probabilities are concluded to be valuable in determining the reliability of the results produced by the given forecast techniques.

This report was written by Dr. Thomas A. Gleeson, Navy Weather Research Facility consultant from Florida State University. Liaison with Dr. Gleeson during the preparation of this report was done by the Task Leader, Mr. S. Donald Case. Mr. Robert S. Haltiner performed the mathematical review and Mr. John M. Mercer the final edit of this report for the Navy Weather Research Facility.

This publication has been reviewed and approved on 10 March 1966 by the undersigned.

JAMES L. KERR
Commander, U.S. Navy
Officer in Charge
U.S. Navy Weather Research Facility

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1. INTRODUCTION

In any large-scale meteorological situation, there is no more than a finite number of synoptic measurements available for map analyses. The lack of information between these reports makes such analyses somewhat uncertain and implies that forecasts based upon them will also be uncertain, to some extent at least. As one meteorologist [8] puts it, "A forecast of weather can never be more than a statement of probabilities."

A second consequence of the inevitable data void between reports is that because small features, which may be significant for predictions, are smoothed out of the analyses, any evaluation of the number or sizes of analysis errors will tend to be underestimated. In other words, we can do no better than to minimize the errors and thereby maximize the estimated probabilities of success of large scale predictions. We conclude that probabilities are *necessary*, and maximum probabilities are *sufficient* in synoptic analysis and forecasting.

The purpose of this publication is to describe a method for obtaining such maximum or limiting probabilities for predictions made by standard quantitative techniques. The method has a theoretical basis, but this will not be emphasized here. In application to standard forecast formulas (extrapolation, barotropic models, etc.), partial derivatives must be taken to arrive at equations needed for probability predictions. But when the latter equations are at hand, their practical use requires nothing more than algebra.

In what follows, no consideration is given to errors in synoptic reports themselves, nor to imperfections of standard forecast techniques. This is another reason why the computed probabilities of success become maximized.

In presentation of the method, it is deemed desirable to use frequent examples, beginning with the simplest. This is done in the next section. A summary of the method is given in chapter 3, and subsequent sections are devoted to discussion and other applications of the general procedure. References [2, 3, 4, 5, and 6] can be consulted for further details.

2. EXAMPLES OF METHOD

2.1 Location of an Isobar

Because of the absence of information be-
tween synoptic observations, the analysis of a
field, sea-level pressure for example, will be
in error to some extent. Figure 2.1 shows an
isobaric analysis from a portion of a hypotheti-
cal pressure field. The 1010-mb. isobar is
drawn through an arbitrary point, A, in this case.
However, unless there happened to be a station
at A reporting 1010 mb., the true pressure there
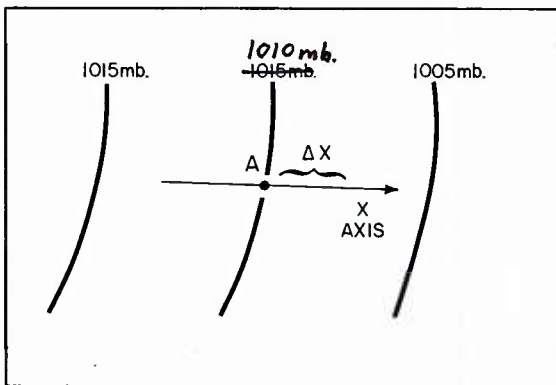probably would not be exactly this value, indi-
cating the isobar to be misplaced.

Figure 2.1. *Example of Isobar Pattern.*

If we define an x axis through A and perpen-
dicular to the isobars, as in figure 2.1, the error
in location of the 1010-mb. isobar can be repre-
sented schematically by the interval Δx. Of
course, the magnitude of Δx cannot be found from
the analysis because continuous observations are
not available. But it is possible to say something
about the probability that a specified magnitude
of Δx will exist.

We can regard the synoptic reports as being
located at random relative to the true pattern
prevailing at a given time in a given geographic
region. (This is not the same as saying the re-
ports are at random, geographically.)

The analysis based on these reports places

the isobar at point A. Now, suppose the x axis
to be fixed relative to the true pattern, and we
consider other locations of the 1010-mb. isobar
along x, as would be obtained from independent
analyses of the same basic pattern but for differ-
ent random arrangements of synoptic reports.
In all such arrangements, assume that the aver-
age density of the observing networks remains
the same. The frequency distribution of many
such locations might be as indicated in figure 2.2.
Here, the horizontal and vertical coordinates are
the x axis and frequency, respectively. The un-
known but true location of the 1010-mb. isobar
is at x^*. Areas under the curve represent prob-
abilities; the total area is unity. Thus, the prob-
ability that the isobar will be located within a
specified interval, $\pm \Delta x$, of its true location, x^*, is
represented by an area on the graph. The shad-
ed area is an example.
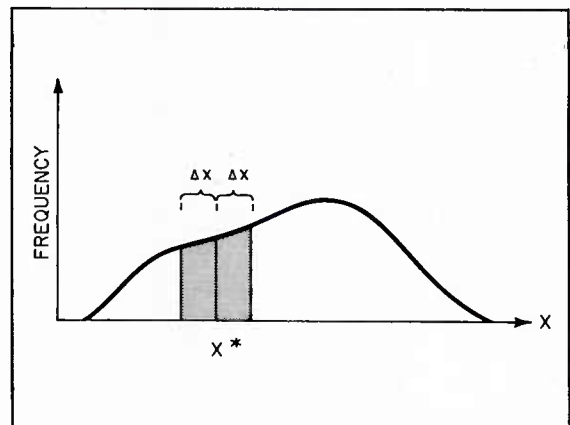
Figure 2.2. *Hypothetically True Frequency Distribution of
Isobar Location.*

Naturally, in practice such a probability
cannot be found because repeated random sam-
plings of a true pattern by observation networks
are not available. Yet, an upper limit to this
probability can be estimated from available in-
formation. This is accomplished by subjecting
the synoptic analysis itself, rather than the true
pattern, to hypothetical random samplings by
networks and evaluating the results. Details of
the sampling theory will not be given here, but
the method for computing limiting probabilities
can be described.

As before, assume that Δx represents the

error in location of an isobar along the perpendicular x axis. With numerous random samplings a large number of such errors is obtained. Now, assume that the mean of the squares of these errors is the variance:

$$\sigma^2(x) = \overline{(\Delta x)^2}, \qquad (1)$$

where the bar, here and hereafter, represents the arithmetic mean of the quantity under it. According to the network sampling theory,

$$\sigma^2(x) = 0.056\, \ell^2, \qquad (2)$$

where 0.056 is a dimensionless constant and ℓ is an average distance between synoptic observations. The latter can be evaluated from the equation,

$$\ell^2 \approx R/n, \qquad (3)$$

where R is the area of the analysis region, and n is the number of synoptic reports in the region. The use of (3) will be descussed further in section 4.1.

Theoretically, $\sigma^2(x)$ is the variance of a normal or Gaussian frequency distribution, as illustrated in figure 2.3. The coordinates are the same as in figure 2.2. The mean of the distribution is $\bar{x}$, with standard deviation σ. Again, areas under the curve represent probabilities, and the total area is unity. The probability that deviations from $\bar{x}$ will not be greater than $\pm \Delta x$ is represented by an area such as the shaded one in this figure.
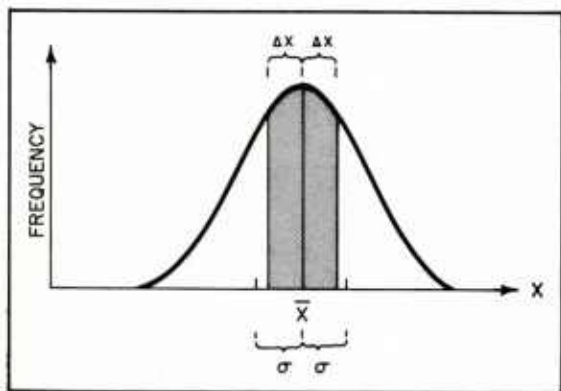
How are these results of use in synoptic analysis? A numerical example will illustrate. Suppose that a sea-level pressure analysis is made over an area of 6,250,000 square miles, approximately the area of South America. There are 100 synoptic pressure reports throughout the region. We want to know something about the accuracy of the analyzed pattern.

With R = 6,250,000 square miles and n = 100, equation (3) yields

$$\ell^2 = 62{,}500\ (miles)^2,$$

or

$$\ell = 250\ miles.$$

Then from (2),

$$\sigma^2(x) = 3{,}500\ (miles)^2,$$

or

$$\sigma(x) = 59.2\ miles.$$

Let Δx = 100 miles. From the ratio

$$\frac{\Delta x}{\sigma(x)} = \frac{100}{59.2} = 1.7.$$

Refer to figure 2.4 which is a graph of $\Delta x/\sigma(x)$ versus probability, derived from tables of the normal distribution [1]. Enter this figure with 1.7 and find that the ordinate value is 91 percent. This is the theoretical probability that at any point the analyzed position of an isobar lies within $\pm$ 100 miles of its hypothetically true position, $\bar{x}$.

Now, compare figures 2.2 and 2.3 which are drawn to the same scale. For the same choice of Δx in both figures, the shaded area in figure 2.3 is larger than the one in figure 2.2, meaning that the corresponding probabilities are unequal in the same sense. When Δx = 100 miles, the larger probability is 91 percent. Theoretically, then, the probability that the analyzed position of an isobar at any point lies within 100 miles of its actual true position, x^*, is less than 91 percent. We do not know where x^* is, but we do have a limiting-probability statement concerning its proximity.

In this example, it was assumed that the probability under the normal curve exceeded that under the "true" curve (fig. 2.2), for the same Δx. This is a general prediction of the network

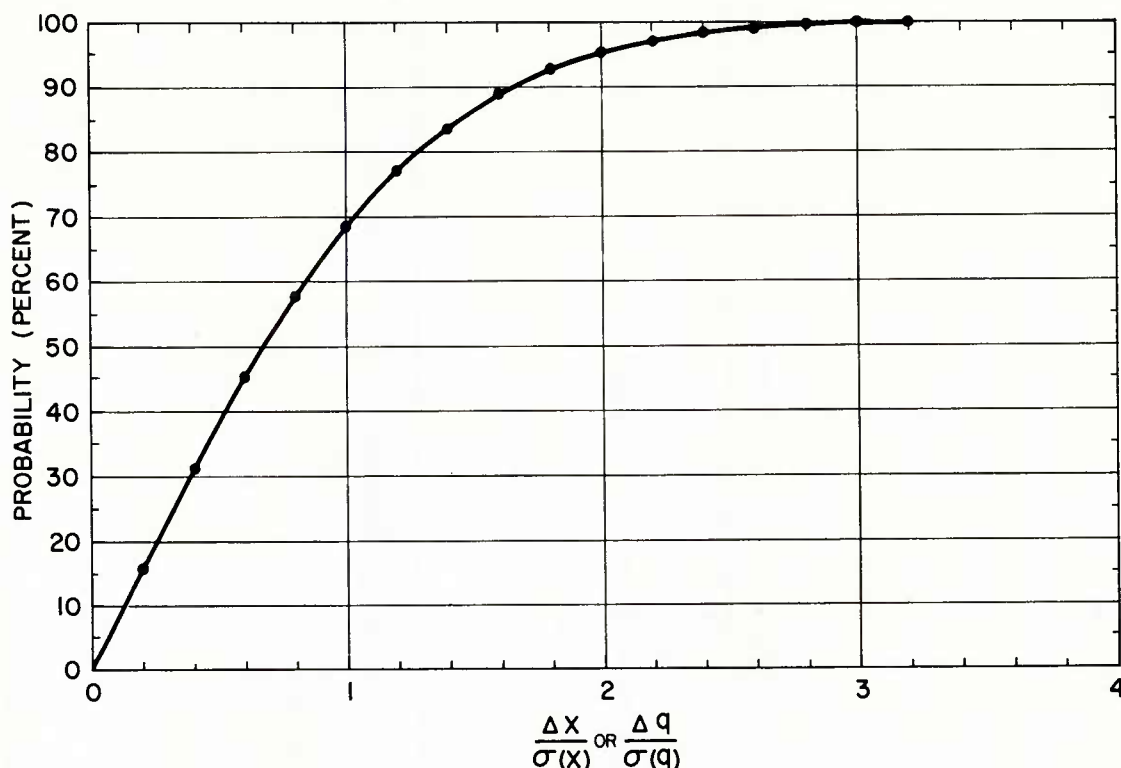Figure 2.3. *Model Frequency Distribution of Isobar Location.*

Figure 2.4. Cumulative Probability Versus Ratios $\Delta x/\sigma(x)$ and $\Delta q/\sigma(q)$.

sampling theory; the limiting probability should be at least as large as the true one. Whether this happens in reality can only be determined by tests. Several tests have been run, all successful so far. Examples will be given later.

It should be noted that the choice of $\Delta x = 100$ miles was arbitrary. Other values are possible; for example, 50 miles. Then

$$\frac{\Delta x}{\sigma(x)} = \frac{50}{59.2} = 0.845,$$

and the limiting probability is 60 percent, as can be verified from figure 2.4.

The accurate location of an isobar may not be of great interest, but equations (1), (2), and (3), and figure 2.4 are fundamental to developments of more interesting and advanced topics discussed below.

2.2 Specification of Pressure

Associated with errors in location of isobars are errors in pressure. For example, at the arbitrary point A in figure 2.1 there is a 1010-mb. analyzed pressure, whereas the true value is probably somewhat different.

What is the relation between pressure error, Δp, and isobar-location error, Δx, at a given point? From the calculus,

$$\Delta p = \frac{dp}{dx} \Delta x; \tag{4}$$

where dp/dx is simply the gradient of p in the x direction. Note that the x axis was chosen along the gradient.

The true gradient of p at point A is not known, but it can be approximated from the analysis. Our procedure now is similar to that of the preceding section; square and average errors to obtain variances. Thus, the variance of p becomes

$$\sigma^2(p) = \overline{(\Delta p)^2} = \left(\frac{dp}{dx}\right)^2 \overline{(\Delta x)^2} = \left(\frac{dp}{dx}\right)^2 \sigma^2(x). \tag{5}$$

In this operation, the value of dp/dx is to be regarded as constant.

As before, $\sigma^2(x)$ is considered to be the variance of a normal distribution. Then because dp/dx is a constant, $\sigma^2(p)$ in (5) is also the variance of a normal distribution, according to statistical theory. Combination of (2) and (5) yields

$$\sigma^2(p) = 0.056 \; \ell^2 \left(\frac{dp}{dx}\right)^2 . \qquad (6)$$

To illustrate the practical use of (6), we employ the example of figure 2.1. Assume that the pressure gradient measured at A from the analysis, is 3 millibars per 100 miles, and that the average distance between synoptic reports again is 250 miles. Then, from (6),

$$\sigma^2(p) = 3.15,$$

or

$$\sigma(p) = (3.15)^{1/2} = 1.77.$$

Select a value of $\Delta p = 2$ mb. Form the ratio

$$\frac{\Delta p}{\sigma(p)} = \frac{2}{1.77} = 1.13.$$

Enter the graph in figure 2.4 with the value 1.13 and find the corresponding probability, 74 percent. This means: *the probability, that the analyzed pressure at A lies within ± 2 mb. of the true pressure at that point, is not greater than 74 percent.* Under the circumstance that synoptic reports are widely scattered, a stronger statement than this hardly seems possible.

2.3 Comparison of Theory and Observation in Pressure Specification

A probability statement like the one above cannot be tested at a single point in an analysis. Observed probabilities are needed for comparison with theory. Because actual synoptic situations do not repeat themselves, observed probabilities at a fixed point are not available and it is necessary to make indirect comparisons, one of which will be described here.

A synoptic situation was selected at random: the sea-level pressure for the entire Northern Hemisphere, 1230 Greenwich mean time, 19 February 1950. There were 718 available synoptic reports of pressure and many of these were accompanied by observed wind values. Of these reports, 353 were deleted on a random basis and the remaining 365 were plotted on a base map and analyzed without prior knowledge of the isobaric pattern. Where available, reported winds aided the analysis. Then the actual error in pressure at each of the 353 deleted stations was found by comparison of analyzed and true values. The theoretical variance, $\sigma^2(p)$, was computed at each of these Locations, by use of (6). (The same value of $\bar{\ell}$, 634 miles, appeared in all computations. This value was found from (3), with $n = 365$ and R the area of the

Northern Hemisphere.)

Actual errors and theoretical variances were combined into separate frequency distributions, but the details of this step are omitted here. Both distributions are shown in figure 2.5. The smooth curve represents a theoretical normal distribution, while the broken curve is a histogram of observed errors. The important feature is that the theoretical curve shows less dispersion from the mean (set at zero) than does the observed curve. In other words, the observed probability of deviations to lie within any specified interval centered about the mean is not greater than the theoretical probability for the same interval. This conclusion supports the network sampling theory, and is typical of results obtained from other similar studies.
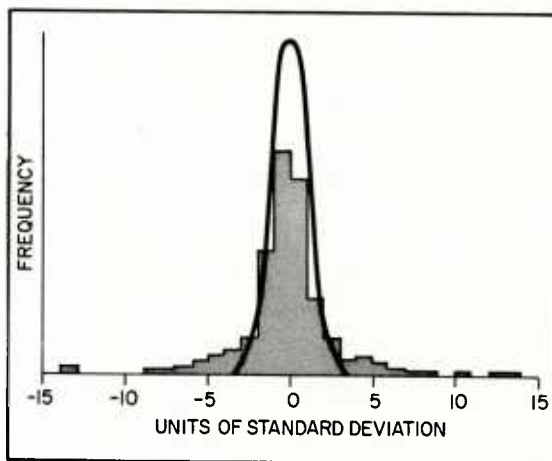
Figure 2.5. *Observed Frequency Distribution (histogram) and Theoretical Frequency Distribution (symmetrical curve) of Pressure-Analysis Errors for Northern Hemisphere, 1230 GMT, 19 February 1950.*

2.4 Extrapolation

In previous sections, the primary concern is the probabilities of accurate synoptic analysis. Henceforth, attention will be focussed on probabilities of accurate predictions from analyses. We begin with simple extrapolation.

Figure 2.6 shows three hypothetical locations of an isobar segment along a perpendicular x axis. The past, present, and extrapolated positions are at x_0, x_1, and x_2, respectively; with distances between positions being equal. Thus,

$$x_2 - x_1 = x_1 - x_0 \qquad (7)$$

Figure 2.6. *Successive Locations of Isobar Segment Along x Axis.*

Because of analysis errors, the displacement $x_1 - x_0$ is somewhat in error, and thereby causes an error in the future displacement, $x_2 - x_1$:

$$\Delta (x_2 - x_1) = \Delta (x_1 - x_0) . \qquad (8)$$

This can be rewritten in terms of the location errors themselves:

$$\Delta x_2 - \Delta x_1 = \Delta x_1 - \Delta x_0 ,$$

or

$$\Delta x_2 = 2 \Delta x_1 - \Delta x_0 . \qquad (9)$$
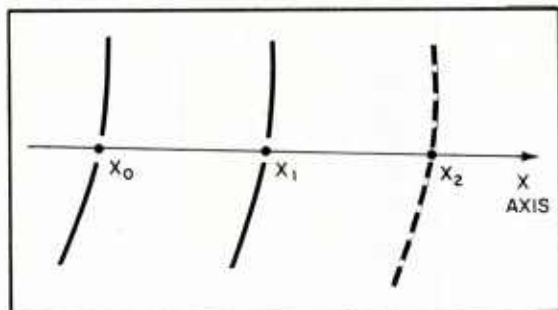
Here, the error in predicted location, Δx_2, is expressed in terms of analysis errors. Now both sides of (9) are squared and averaged to obtain $\sigma^2(x_2)$, the variance of prediction errors. Thus,

$$\sigma^2(x_2) = \overline{(\Delta x_2)^2} = 4 \overline{(\Delta x_1)^2} + \overline{(\Delta x_0)^2} - 4 \overline{\Delta x_1 \Delta x_0} . \quad (10)$$

We will assume that present and past synoptic analyses were made independently of each other, or else that the time interval between successive maps is large enough so that errors Δx_0 and Δx_1 are essentially independent of each other. This makes it possible to set the cross-product term $\overline{\Delta x_1 \Delta x_0} = 0$, as the two errors are uncorrelated. In practice, either of these assumptions is not a severe restriction. And it is most important to eliminate $\overline{\Delta x_1 \Delta x_0}$ from (9), otherwise it cannot be evaluated. See section 4.2 for further discussion of this subject.

Next, $\overline{(\Delta x_1)^2}$ and $\overline{(\Delta x_0)^2}$ are each replaced with $0.056 \, \ell^2$, from (1) and (2), so that (10) becomes

$$\sigma^2(x_2) = 0.056 \, \ell^2 \, (4 + 1) = 0.280 \, \ell^2 . \qquad (11)$$

Now, suppose $\ell = 250$ miles as before. From (11),

$$\sigma^2(x_2) = 17,500 \, (miles)^2$$

or

$$\sigma(x_2) = 132.29 \, miles.$$

Select an interval $\Delta x_2 = 100$ miles. Form the ratio

$$\frac{\Delta x_2}{\sigma(x_2)} = \frac{100}{132.29} = 0.76 .$$

With figure 2.4, this yields 55 percent probability. *In a network whose average distance between stations is 250 miles, the probability that an isobar extrapolated by use of equation (7) will lie within ± 100 miles of its true location is 55 percent, at best.*

Two features of this example should be noted. It has been implicitly assumed that extrapolation is a perfect prediction method, because no method errors were considered. However, the italicized statement above still holds, even if extrapolation is a poor procedure.

Secondly, the results are not restricted to isobar displacements. Thus, x_2 could represent the extrapolated position of a front, isotherm, or other synoptic feature.

2.5 Pressure Forecast Using Pressure Tendency

The pressure at a fixed point can be forecast from the relation,

$$p_t = p_0 + b_0 t , \qquad (12)$$

where t is time; b_0 the initial tendency, $\partial p / \partial t$; and p_0 and p_t are initial and forecast pressures, respectively. In figure 2.7, let A be such a point, and let x_1 and x_2 axes be drawn perpendicular to isobars (solid curves) and isallobars (dashed curves) through the point, as an example. The forecast error, Δp_t, depends on analysis errors, Δp_0 and Δb_0. Thus,

$$\Delta p_t = \Delta p_0 + t \Delta b_0 . \qquad (13)$$

From (4),

$$\Delta p_0 = \frac{dp_0}{dx_1} \Delta x_1 , \qquad (14)$$

and, by similar reasoning,

$$\Delta b_0 = \frac{db_0}{dx_2} \Delta x_2 , \qquad (15)$$

where db_0/dx_2 is the isallobaric gradient. Substitution of (14) and (15) into (13) yields

$$\Delta p_t = \frac{dp_0}{dx_1} \Delta x_1 + t \frac{db_0}{dx_2} \Delta x_2 . \qquad (16)$$

The variance of forecast errors, $\sigma^2(p_t)$, is found by squaring and averaging both sides of (16):

$$\sigma^2(p_t) = \overline{(\Delta p_t)^2} = (\frac{dp_0}{dx_1})^2 \overline{(\Delta x_1)^2} + t^2 (\frac{db_0}{dx_2})^2 \overline{(\Delta x_2)^2} \\ + 2t \frac{dp_0}{dx_1} \frac{db_0}{dx_2} \overline{\Delta x_1 \Delta x_2} . \qquad (17)$$
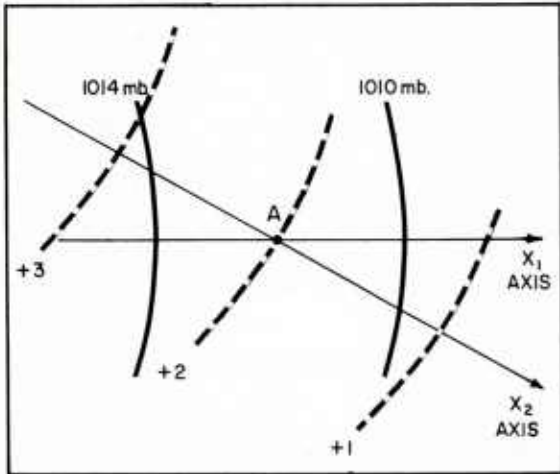

Figure 2.7. Example of Isobar (solid lines) and Isallobar (dashed lines) Patterns.

Here, dp_0/dx_1 and db_0/dx_2 are treated as constants to be evaluated from the synoptic analysis, and the last term is set equal to zero because analysis errors of isobars and isallobars are mutually independent ($\overline{\Delta x_1 \Delta x_2} = 0$).

For a numerical example of (17), let $\ell = 250$ miles, $dp_0/dx_1 = 3$ mb./100 miles, and $db_0/dx_2 = 1$ mb./3 hr. per 100 miles. Then, as in equation (11),

$$\sigma^2(p_t) = 0.35 (9 + 0.11 \, t^2) , \qquad (18)$$

where t is measured in hours.

Let $\Delta p_t = 2$ mb. By use of (18) and figure 2.4, limiting probabilities, P, for p_t can be found, as in previous sections. Results for various values of t are given in table 2.1. (The last column shows values of p_t predicted from (12), with assumed initial values, $p_0 = 1012$ mb. and $b_0 = +2$ mb./3 hr., at point A.) Limiting probabilities of successful prediction of pressure within ± 2 mb., decrease with increasing forecast periods, which is consistent with experience.

Table 2.1. Predicted Pressures (p_t) and Limiting Probabilities (P) for Pressures to Be Within 2 mb. of True Values at Various Times (t).

t (hour)	$\frac{2}{\sigma(p_t)}$	P (percent)	p_t (mb.)
0	1.128	74	1012
6	0.936	65	1016
12	0.676	50	1020
18	0.504	39	1024
24	0.396	31	1028

3. SUMMARY OF METHOD

With the examples of the previous section as an introduction, we can outline the method for obtaining limiting probabilities quite generally and formally. Some comments about the example in section 2.5 are appended to illustrate each step:

1. Select variable, q, to be studied.

2. Express this variable as a function of other variables.

3. Obtain total differential of the variable in terms of other variables.

4. Simplify total differential by omitting from it any difference terms not due to synoptic analysis errors.

5. Repeat steps 2, 3, and 4 for each of the other variables in the total differential. Continue the repetition for all new variables introduced during the process, as far as possible.

6. Substitute all total differentials obtained in step 5 into first total differential. This gives the *basic error equation*.

7. Square and average both sides of the basic error equation. This gives the *variance equation*.

8. Set equal to zero the cross-product terms that are averages of products of uncorrelated errors.

9. Replace each $\overline{(\Delta x_i)^2}$ with $0.056 \; \ell^2_i$.

10. Evaluate ℓ^2 from R/n, and partial derivatives from ratios of finite differences, using data from synoptic analyses.

11. Use computed value of $\sigma(q)$ and specified value of Δq to find limiting probability from figure 2.4.

Comments:

1. In the example of section 2.5, we let $q = p_t$.

2. According to (12), p_t is a function of p_0, b_0, and t :

$$p_t = p_t (p_0, b_0, t) . \qquad (19)$$

3. From the calculus and (19), the total differential is written

$$\Delta p_t = \frac{\partial p_t}{\partial p_0} \Delta p_0 + \frac{\partial p_t}{\partial b_0} \Delta b_0 + \frac{\partial p_t}{\partial t} \Delta t . \qquad (20)$$

The partial derivatives in (20) can be found by differentiation of (12). Thus,

$$\frac{\partial p_t}{\partial p_0} = 1, \; \frac{\partial p_t}{\partial b_0} = t, \frac{\partial p_t}{\partial t} = b_0 . \qquad (21)$$

When (21) is substituted into (20), the latter becomes

$$\Delta p_t = \Delta p_0 + t \Delta b_0 \mp b_0 \Delta t . \qquad (22)$$

4. The last term in (22) can be omitted because there is no error in t introduced by synoptic analysis; the forecast period can be specified accurately, independently of any analysis. Therefore. $\Delta t = 0$ and (22) reduces to (13).

5. The other variables in (13) are p_0 and b_0. Their total differentials are given simply by (14) and (15). The new variables introduced by (14) and (15) are x_1 and x_2. But, the latter are independent variables, so the repetition of steps 2, 3, and 4 terminates here.

6. This step results in (16).

7. This gives (17).

8. The term, $2t \dfrac{dp_0}{dx_1} \dfrac{db_0}{dx_2} \overline{\Delta x_1 \Delta x_2}$, is dropped from (17).

9 and 10. These steps lead to (18).

11. See table 2.1.

4. DISCUSSION

4.1 Evoluation of ℓ

According to (3), the square of the average distance between synoptic reports is

$$\ell^2 = R/n \ ,$$

where R is the geographic area over which the analysis is made, and n is the number of reports in that area. In all studies to date, it has been satisfactory to compute ℓ in this way. For example, the successful results described in section 2.3 were obtained by letting R be the total area of the Northern Hemisphere and n the number of hemisphere stations used in analysis.

A special problem arises when analyses are made over oceans where data tend to be sparse. If data are too sparse, no reliable analyses can be made. If there are some data permitting analyses of large-scale features, at least, then R should be made the ocean area although n is a small number. However, if the weather systems to be analyzed have emerged from a nearby land area having a dense observation network, so recently that by continuity the analyst still has a good concept of their features, then it is suitable for computation purposes to use a value of ℓ characteristic of the dense land network. It is important here to realize that the chief goal is a limiting or maximized probability value, and that this is not unique in a given problem. This is because all probabilities, from unity down to computed limiting value, are themselves limiting probabilities. Of course, it is desirable to obtain a value as close to a lower "true" probability as possible. But at least it can be ensured that a probability will be a limiting one if ℓ is made sufficiently small. Network densities on land are suitable for this purpose.

4.2 Treatment of Cross-Product Terms

This subject arose earlier. In section 2.5, the errors of isobaric and isallobaric analyses were regarded as independent of each other, which is a reasonable assumption, so that the term containing $\overline{\Delta x_1 \Delta x_2}$ in (17) could be set equal to zero.

In the extrapolation example of section 2.4, the justification for dropping $\overline{\Delta x_1 \Delta x_0}$ from (10) is that the analyses at different map times are made independently of each other or that the interval between these maps is large enough to guarantee independence of errors. An apparently safe guide in practice is that the map interval, τ, satisfies the inequality

$$\tau \geq \ell / c_{max} \ , \tag{23}$$

where c_{max} is the maximum apparent speed that synoptic features (isobars, fronts, etc.) are observed to move. Six hourly and twelve hourly maps satisfy this criterion in dense networks, particularly on land.

The problem of cross-product terms also is present when evaluating errors of gradients. For example, suppose it is desired to study errors of the east-west component of the geostrophic wind, u, on a constant pressure surface. Now,

$$u = -\frac{g}{f} \frac{\partial Z}{\partial y} \ , \tag{24}$$

where g is the acceleration of gravity, f the Coriolis parameter, and $\partial Z / \partial y$ is the north-south height gradient of the constant pressure surface. In practice, the gradient would be computed as a ratio of finite differences:

$$\frac{\partial Z}{\partial y} \simeq \frac{Z_2 - Z_1}{L} \ , \tag{25}$$

where L is a selected distance along the y axis centered at the point where u is to be found, and Z_1 and Z_2 are heights interpolated from the contour analysis at end points of L. Then (24) becomes

$$u = -\frac{g}{f} \frac{(Z_2 - Z_1)}{L} \ . \tag{26}$$

Following the general method in chapter 3, we regard u as a function of g, f, Z_1, Z_2, and L and write its total differential as

$$\Delta u = \underline{\frac{\partial u}{\partial g} \Delta g} + \underline{\frac{\partial u}{\partial f} \Delta f} + \frac{\partial u}{\partial Z_1} \Delta Z_1 + \frac{\partial u}{\partial Z_2} \Delta Z_2 + \underline{\frac{\partial u}{\partial L} \Delta L} \ . \tag{27}$$

The underlined terms become zero because g is a known constant, L is predetermined exactly, and f depends on latitude and is assumed constant. By partial differentiation of (26), (27) reduces to

$$\Delta u = -\frac{g}{f} \frac{\Delta Z_2}{L} + \frac{g}{f} \frac{\Delta Z_1}{L} = -\frac{g}{f} \frac{(\Delta Z_2 - \Delta Z_1)}{L} \ . \tag{29}$$

The total differentials of Z_1 and Z_2 are

$$\Delta Z_1 = \frac{\partial Z_1}{\partial x_1} \Delta x_1$$

and

$$\Delta Z_2 = \frac{\partial Z_2}{\partial x_2} \Delta x_2 \quad , \qquad (29)$$

where axes x_1 and x_2 are defined to be along the height gradients at the points where Z_1 and Z_2 are interpolated from the analysis. By combination of (28) and (29), we have

$$\Delta u = -\frac{g}{fL} \left(\frac{\partial Z_2}{\partial x_2} \Delta x_2 - \frac{\partial Z_1}{\partial x_1} \Delta x_1 \right) \qquad (30)$$

The variance of u is obtained by squaring and averaging (30):

$$\sigma^2(u) = \overline{(\Delta u)^2} = \left(\frac{g}{fL}\right)^2 \left[\left(\frac{\partial Z_2}{\partial x_2}\right)^2 \overline{(\Delta x_2)^2} + \left(\frac{\partial Z_1}{\partial x_1}\right)^2 \overline{(\Delta x_1)^2} \right.$$
$$\left. - 2 \frac{\partial Z_1}{\partial x_1} \frac{\partial Z_2}{\partial x_2} \overline{\Delta x_1 \Delta x_2} \right] \quad . \qquad (31)$$

The last term is a cross product that can be set equal to zero if Δx_1 and Δx_2 are uncorrelated. The latter condition will be met if the distance chosen for L is sufficiently large, according to the inequality

$$L \geqq \ell \quad . \qquad (32)$$

This is not a severe restriction in practice. Then with the aid of (1) and (2), (31) becomes

$$\sigma^2(u) = 0.056 \, \ell^2 \left(\frac{g}{fL}\right)^2 \left[\left(\frac{\partial Z_1}{\partial x_1}\right)^2 + \left(\frac{\partial Z_2}{\partial x_2}\right)^2 \right] \quad , \qquad (33)$$

which can be evaluated with the aid of (3).

As a final instance wherein cross products appear but can be eliminated, consider the problem of errors in determination of thickness between two isobaric surfaces. Let h be the thickness between the 700-mb. and 1000-mb. surfaces at a given geographic location, so that

$$h = Z - z \quad , \qquad (34)$$

where Z and z are the heights of the upper and lower surface, respectively. By standard procedure,

$$\Delta h = \Delta Z - \Delta z = \frac{\partial Z}{\partial x_1} \Delta x_1 - \frac{\partial z}{\partial x_2} \Delta x_2 \quad , \qquad (35)$$

where the x_1 and x_2 axes lie along the height gradients at 700 and 1000 mb. Then,

$$\sigma^2 (h) = \overline{(\Delta h)^2} = \left(\frac{\partial Z}{\partial x_1}\right)^2 \overline{(\Delta x_1)^2} + \left(\frac{\partial z}{\partial x_2}\right)^2 \overline{(\Delta x_2)^2}$$
$$- 2 \frac{\partial Z}{\partial x_1} \frac{\partial z}{\partial x_2} \overline{\Delta x_1 \Delta x_2} \quad . \qquad (36)$$

Now, if the contour analyses have been made independently we can set $\overline{\Delta x_1 \Delta x_2} = 0$, particularly when h is large enough (as in this case) to be of practical value or interest. The variance equation becomes

$$\sigma^2(h) = 0.056 \, \ell^2 \left[\left(\frac{\partial Z}{\partial x_1}\right)^2 + \left(\frac{\partial z}{\partial x_2}\right)^2 \right] \qquad (37)$$

after the cross product term is dropped.

4.3 Presentation of Results

Each of the previous examples is concerned with a single probability value at an initial or forecast time. Depending on the meteorological variable, it may be possible to obtain a field of predicted probabilities associated with the field of the predicted variable. Thus, suppose the height, Z, of the 500-mb. surface is to be forecast at numerous grid points by a standard method. Then, at each point the forecast variance $\sigma^2(Z)$ can be obtained and used to find a limiting probability there. The grid field then can be analayzed with isolines of probability.

Figures 4.1 to 4.3 provide an example. The first figure shows the analyzed height field at 500 mb., 0000 Greenwich mean time, 8 January 1963. By means of a barotropic model which conserves absolute vorticity, a numerical 24-hour forecast was made from this initial state, and is shown in figure 4.2. Then, the method of chapter 3 was applied to the barotropic-model equation and the synoptic features of figure 4.1 to yield numerical predictions of $\sigma^2(Z)$ at grid points throughout the North American region. Corresponding, limiting probabilities were plotted and analyzed in figure 4.3. The value of each isopleth is the limiting probability for predicted height values to lie within $\pm$ 100 feet of true values at 0000 Greenwich mean time, 9 January 1963. The map shows areas where forecasts have good and poor likelihoods of success.

At this point the reader may have the following question in mind. Suppose two analysts independently drew separate contour fields for the synoptic situation of January 8. Their analyses would be slightly different, consequently their analyzed probability fields for January 9 would differ from each other to some extent. Would both fields still give valid limiting probabilities?

From tests so far and from deliberate intent in development of the method, the answer is yes. Synoptic analyses are gross smoothings of reality that presumably resemble each other more closely than any one of them would resemble the true situation in all its complexity.
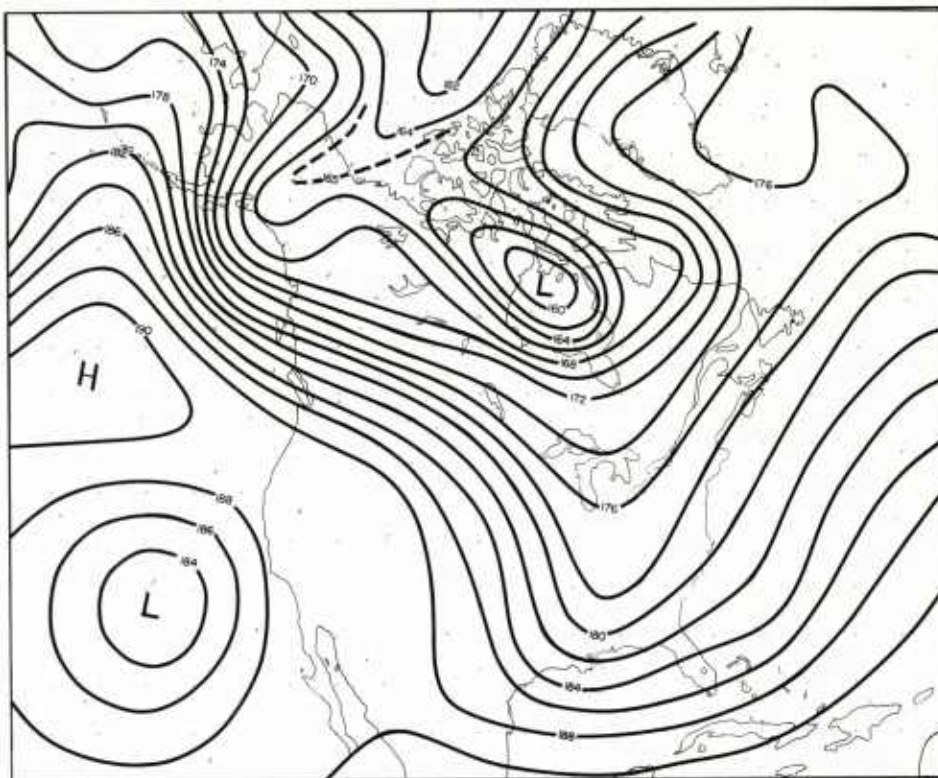
Figure 4.1. Initial Height Field (100's of ft.) at 500 mb., 0000 GMT, 8 January 1963.
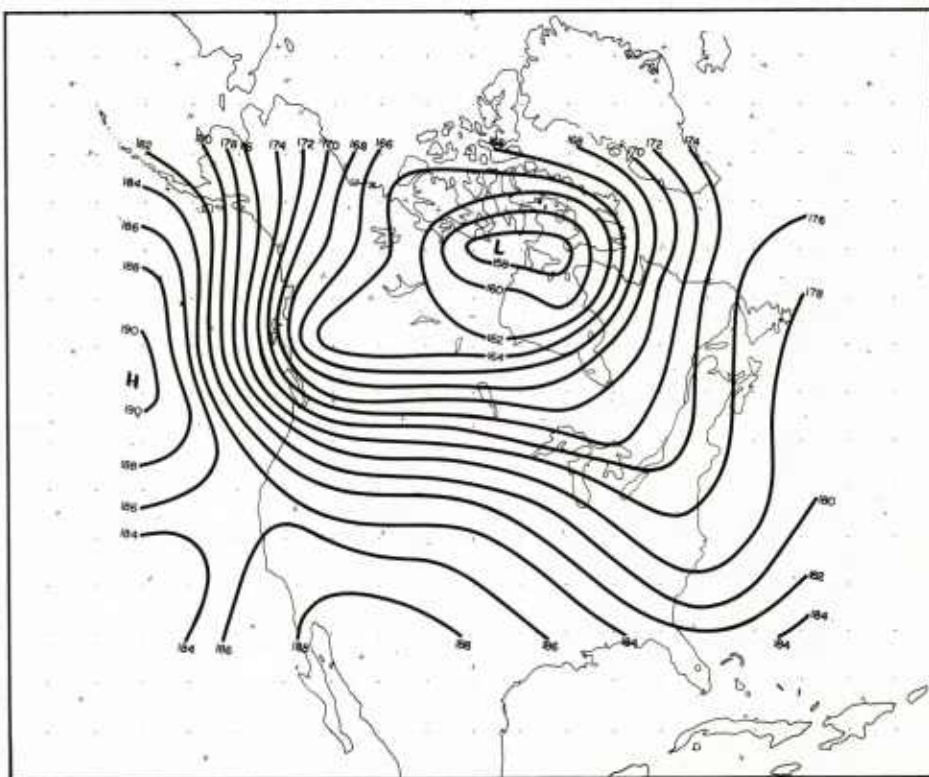

Figure 4.2. Forecast Height Field (100's of ft.) at 500 mb., 0000 GMT, 9 January 1963.
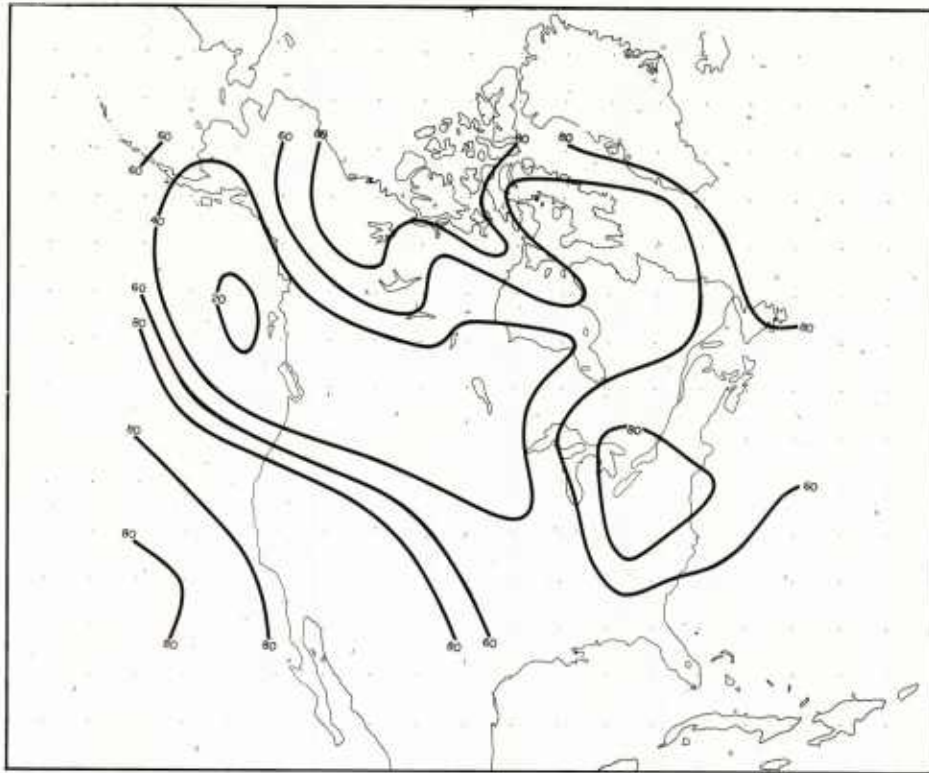
Figure 4.3. Forecast Probability Field (percent) at 500 mb., 0000 GMT, 9 January 1963.

4.4 Verifications

A comparison between observed and theoretical probabilities was given earlier in figure 2.5. Another type of comparison which has a simple interpretation, will be described in this section.

Suppose that forecast and verification height fields at the 500-mb. level, at a specified time, are to be compared with each other. At each of N points, the error in the forecast height can be computed. A frequency distribution of the N_i errors will then reveal what fraction fell within ± 100 feet of verification values, for example.

Next, the forecast variance, $\sigma^2(Z)_i$, at each of the N points can be used to find the limiting probability, P_i, for the predicted height to fall within ± 100 feet of the true height. Then one can determine the average (expected) probability, $\bar{P}$ for this result at the N_i points, from

$$\bar{P} = \frac{1}{N}(P_1, P_2, \ldots, P_i, \ldots, P_N) \ . \qquad (38)$$

If N_i is large (greater than 100), and $\bar{P}$ equals or exceeds the observed fraction of errors, this may be accepted as support for the theoretical probability.

Figure 4.4 shows an actual comparison between theoretical probabilities (T) and observed relative frequencies (0) for various error intervals (ΔZ). This is a verification graph for 10 synoptic cases in which Z was predicted by the numerical barotropic model mentioned in the last section. For each value of ΔZ, $N = 1,645$
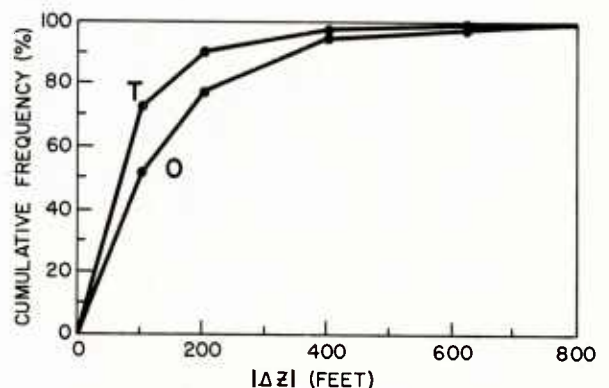

Figure 4.4. Observed (0) and Theoretical (T) Limiting Probabilities for Forecast Heights to Be Within Interval ΔZ of True Heights at 500 mb., Based on 1645 Numerical Forecasts in 10 Synoptic Situations.

pairs of values were used. It is apparent that the theoretical probabilities are truly limiting values because they bound the observed values from above.

Parenthetically, it may be noted that the predicted variances, $\sigma^2(Z)$, can also be used to determine limiting probabilities of predicted geostrophic wind components. Thus, for the u component, the variance $\sigma^2(u)$ results from squaring and averaging (28), to give

$$\sigma^2(u) = \overline{(\Delta u)^2} = \left(\frac{g}{fL}\right)^2 \left[\overline{(\Delta Z_2)^2} + \overline{(\Delta Z_1)^2}\right]$$

$$= \left(\frac{g}{fL}\right)^2 \left[\sigma^2(Z_2) + \sigma^2(Z_1)\right], \qquad (39)$$

from which the cross product term has been suitably eliminated. Graphs similar to that of figure 4.4 have been computed for u and v components, and with similar success.

5. APPLICATION TO EXTRAPOLATION FORMULAS

Given below are five equations for extrapolating synoptic features, and corresponding variance equations for predicted displacements. Each method refers to figure 5.1 which shows three consecutive past positions and the extrapolated position of an isoline (A, B, C, and X, respectively along an x axis. It is not necessary that this axis be straight, as shown, but it should be perpendicular to the isoline at all four intersection points.
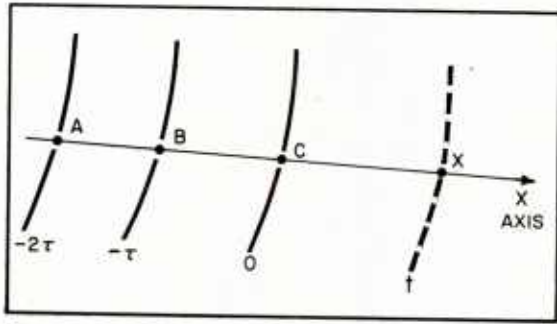

Figure 5.1. *Successive Locations of Synoptic Feature Along x Axis.*

5.1 Constant Speed

$$x_1 = C + vt \ , \tag{40}$$

where

$$v = \frac{C - B}{r} \tag{41}$$

is the measured speed from B to C in map interval r, and t is time. Then

$$\sigma^2(x_1) = 0.056 \, \ell^2 \left[1 + 2\frac{t}{r} + 2 \left(\frac{t}{r}\right)^2 \right] \tag{42}$$

is the variance equation for x_1.

5.2 Variable Speed

$$x_2 = C + vt + \frac{1}{2} at^2 \ , \tag{43}$$

where v is given by (41), and

$$a = \frac{1}{r} \left(\frac{C - B}{r} - \frac{B - A}{r} \right) \tag{44}$$

is the measured acceleration involving distances

B–A and C–B , and there is the same time interval, r , between successive maps. The variance equation takes the form

$$\sigma^2(x_2) = 0.056 \, \ell^2 \left[1 + 2\frac{t}{r} + 3 \left(\frac{t}{r}\right)^2 + 3 \left(\frac{t}{r}\right)^3 \right. $$
$$\left. + 1.5 \left(\frac{t}{r}\right)^4 \right] \ . \tag{45}$$

5.3 Percentual Change Method for $t = r$

$$x_3 - C = (C - B)^2/(B - A) \ . \tag{46}$$

$$\sigma^2(x_3) = 0.056 \, \ell^2 \, (1 + 4k + 8k^2 + 4k^3 + 2k^4) \ , \tag{47}$$

where

$$k = (C - B) /(B - A) \ , \tag{48}$$

here and hereafter.

5.4 Percentual Change Method for $t = 2r$

$$x_4 - x_3 = (x_3 - C)^2/(C - B) \ ; \tag{49}$$

$$\sigma^2(x_4) = 0.056 \, \ell^2 \, (1 + 8k + 32k^2 + 56k^3 $$
$$+ 52k^4 + 24k^5 + 8k^6) \ . \tag{50}$$

5.5 Wasko's Method

$$x_5 - C = C - B + \frac{B - A}{C - B} \left[(C - B) - (B - A) \right] \ , \tag{51}$$

under the condition that $\quad C - B \geqq B - A$.

$$\sigma^2(x_5) = \frac{0.056 \, \ell^2}{k^4} \, (2 + 4k + 12k^2 - 4k^3 $$
$$+ 5k^4) \ . \tag{52}$$

References [5, 7, and 9] can be consulted for discussions of equations (40) through (52). The reader may find it useful practice to verify the variance equations by the procedure of chapter 3.

For practical purposes, the variance equations have been graphed: (42) and (45) in figure 5.2; (47), (50), and (52) in figure 5.3. Each figure is entered along the horizontal axis to find a value of the ratio $\sigma(x)/\ell$ on the vertical axis. Then, with a value of ℓ from (3), $\sigma(x)$ can be found.

For an example, consider the extrapolation case in section 2.4. Actually, this is a special case of method 5.1 when $t = r$. Therefore, enter figure 5.2 at the point $(t/r) = 1$ and find from curve x_1 that $(\sigma/\ell) = 0.53$. Let $\ell = 250$ miles,

then $\sigma = 132.5$ miles, which is close to the value 132.29 miles, found in section 2.4.
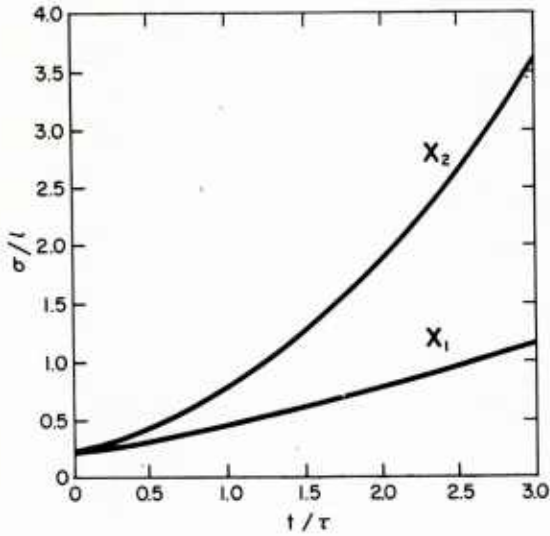

Figure 5.2. Values of σ/ℓ for Predicted Displacements x_1 and x_2 as Functions of t/τ.
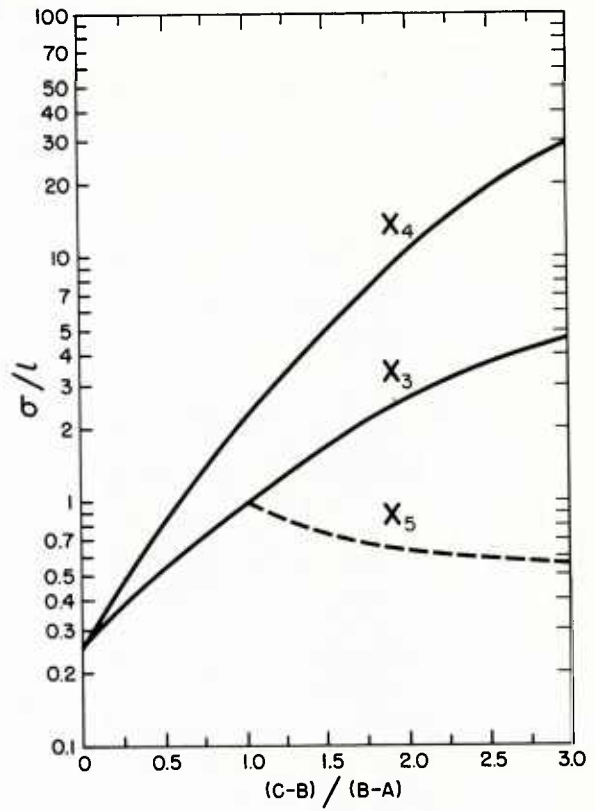

Figure 5.3. Values of σ/ℓ for Predicted Displacements x_3, x_4, and x_5 (dashed line) as Functions of $(C-B)/(B-A)$.

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